Prerequisite Skills
and
Mathematics Learning

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Prerequisite Skills and Mathematics Learning[1]

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1 Excerpted from
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Toys and games are synonymous with play, pleasure, and relaxation. Almost everyone likes to play and, in one form or other, this continues throughout one's life. Play is not just a filling in of an empty period or a relaxation or leisure activity, but it is also an important learning experience—an essential ingredient for growth and development for children and adults alike. For example, babies play with their fingers and toes and in so doing bring about a social interaction with adults who join in their game. As the baby explores this form of play, it helps the child acquire the prerequisite skills needed to learn numeracy.

In addition to the commonly accepted means of relaxation, play is seen as a means to work off aggression, to learn basic survival skills (as is also observable in the animal kingdom), and to learn social behavior (competitive and cooperative). But more importantly, the role of play is to engage in learning and to gain interest in learning.

The purpose of early childhood experience during the Pre-Kindergarten and Kindergarten is develop:

(a) Neuro-psycho-physiological maturation,
(b) Socio-linguistic maturation,
(c) Quantitative reasoning, and
(d) Spatial orientation and space organization.

Development in all of these areas is facilitated by formal and organized learning experiences, but formal and informal play, toys and games have an important role in it.

Fun is a great motivator for learning any subject, even mathematics. Unfortunately, many in our society feel inadequate even when it comes to mundane, everyday calculations, and they view mathematics as an anxiety-provoking task. That makes it difficult for teachers to teach mathematics and for students to invest the interest, time, and energy it takes to learn mathematics. Fortunately, through games and toys, we have the motivational and fun factor that helps move learning along.

Most children come to Kindergarten able to recite the alphabet fluently, in the case of English language, the twenty-six letters. In many countries that number is much larger. Although this is just a rhyme for children at this stage, but, it is the beginning of the development of language containers for sounds and therefore beginning reading. However, very few children enter children able to recite the numbers up to thirty in sequence. Parents need to pay the same kind of attention for numbers as they do for letters. In many countries, parents do often chant a rhyme such as the one below while touching each of the child's toes:

one, two, three, four, five,
once I caught a fish alive:
six, seven, eight, nine, ten,
then I let him go again.

This is the beginning of parents' efforts towards assisting the child to learn to count.

But, it needs to go beyond just the first ten numbers.

In the information and Internet age, smart-phones, computer, and web-based games have become a major source of play among children. They rapidly learn how to operate these devices because they are motivated to explore. Children's sense of adventure through computer and web-based games supports learning in the classroom and laboratory when youngsters progress to other subjects, for example, science in secondary school. These games provide opportunities for acquiring many skills for learning (visuo-spatial, sequencing, and spatial sense). For a fuller development of the brain, though, and for broader learning and sociolinguistic skills, multi-sensory games are essential because they develop and integrate tactile/kinesthetic, visual, auditory, socio-linguistic, and executive function skills.

These days, board games are not limited to play dates and family game nights. Classic games like Scrabble, Candy Land, Go Fish, and Sorry are finding their way into classrooms as educators creatively use many of the popular games to reinforce mathematics, language and critical thinking skills. Numerous research studies support the assertion that playing board games helps students improve mathematics and thinking skills. For example, in one study, disadvantaged preschool students played a simple numeric board game four times for 15-20 minutes at a time over a two-week period. At the end of the two weeks, researchers found students' knowledge of math greatly increased in several different areas related to quantitative thinking and number sense. Number sense is a form of quantitative thinking—knowing what a collection representing the number 5 looks like and knowing that 5 is less than 8 and that 8 is made up of 5 and 3. That does not come just through counting.

Number concept, therefore, numbersense depends on counting, learning quantitative language, and understanding the spatial representation and distribution of objects in the representation of a number, but counting alone is not adequate for developing the true number concept and numbersense. Counting is only a part of number conceptualization process—it is just the beginning. It takes several developmental steps to attach number value to collections and amounts and giving meaning to numbers, and then understanding interrelationships of numbers.

Number concept is the integration of several prerequisite skills: acquiring number names, sequencing, one-to-one correspondence, visual cluster—arrangement of objects, and decomposition/re-composition of visual clusters (numbers)—can the child break the number (from 2 to 10) into its component sub-clusters (smaller numbers) as needed (e.g., 10 is made up of 1 and 9; 2 and 8; 3 and 7; 4 and 6; and 5 and 5 instantly).

Number conceptualization is easily achieved by games and toys. For example, games involving playing cards, dominoes, dice bring the essential skills—sequencing, one-to-one correspondence, visual clustering, and decomposition/recomposition, together. Many card and board games reinforce number concept and numbersense and help children learn these pre-requisite skills but most importantly logical reasoning and communication of ideas.

The benefits of using board games are not limited to mathematics. They can be used to build vocabulary, spelling, and logical reasoning skills. Basically any game can be adapted to help learn. For example, the card game ***Go Fish*** can be modified into a game ***Go Make Ten, Go Makes Eleven, Go Make*** **____**, etc. This is a series of games to efficiently develop the 45 ***sight facts*** (the sums of numbers up to ten). It is played in the same way as **Go Fish**, but here children make as many pairs of two cards that add to ten, or any other number you want children to practice its sight facts. In a short time, children learn the most important arithmetic facts clusters: what two numbers make ten? Children who do not have mastered this family of addition facts up to 10 have great deal of difficulty in mastering other addition facts (a non-negotiable skill at first grade). The game can further be modified to other number facts: ***Go Make Nine***, ***Eight***, or any other number. Because of their intrinsic entertainment value, board games provide educators with an effective tool for engaging students. Games facilitate a welcoming learning atmosphere because students just think they're having fun.

Many parents want to work with their children to help them in mathematics, but they may feel that they have limited mathematics training and understanding of mathematical concepts, particularly the "modern math" or "new ways" of teaching mathematics. Other parents may be impatient with children having problems because they have high expectations of their children. While working with their children, they may also be mixing the roles of parenting and teaching, which can cause difficulties both in learning and personal relationships. To avoid transferring their own anxieties or setting unrealistic expectations, parents should work on mathematics only if they feel comfortable with mathematics and have realistic goals for their children.They should also avoid mixing the two roles: parent and teacher. IN the setting of games, they can achieve these goals.

Despite these possible limitations, there is a great deal parents can do to help children in learning mathematics. For example, they can help children acquire prerequisite skills for mathematics learning. There are certain non-mathematical skills that are obvious for learning mathematics, they are: 1) Memory (to learn basic terminology and hold information in the mind's eye, visualization, working memory), 2) Inductive thinking (to see patterns, going from specific examples to generic rules), 3) Deductive thinking (to have the ability to apply generic rules to specific problems, 4) Spatial orientation/space organization, and 5) Task Analysis (the ability to break down a given problem into smaller, manageable problems). However, there are several other prerequisite skills that help children learn, retain, and master formal concepts, skills, and procedures in mathematics. In any instructional or remedial program, whether the student is eight years old, fifteen, or even an adult, we need to devote a portion of the teaching time in helping the student acquire prerequisite skills such as:

- Matching/one-to-one correspondence with sequence
- Classification/class inclusion
- Visual clustering
- Ordering and sequencing
- Visualization
- Ability to follow sequential directions
- Spatial orientation and space organization
- Estimation
- Pattern recognition, extension, and its application
- Deductive and inductive thinking

Games and toys can help children acquire prerequisite skills for mathematics learning and can better prepare them for all kinds of learning. To develop prerequisite skills successfully, games and toys should have certain characteristics:

1. Games should be based on strategies, not on luck. In other words, to be proficient in a game should mean proficiency in the game's strategies. This means that each encounter with the game or toy helps the child discover something more about the game, i.e., a strategy, a perspective, or a relationship between moves. Such games are interesting to novice and expert alike and help children improve their cognition, inquisitiveness, perseverance, visualization, and executive functions.
2. In general, a game should last on an average ten to fifteen minutes so that children can see the end of the game in a fairly short period of time. This helps them understand the relationship between a strategy and its impact on the game. This teaches children the foundation of deductive thinking or what can be understood as cause and effect. Only when a child has more interest and maturity and is able to handle delayed gratification are complex strategy games such as chess are meaningful. For some children, games like chess may become end in themselves, which is fine, but then they no longer serve the same purpose that we advocate—preparation for prerequisite skills for mathematics learning. For such a student mathematics learning is not difficult.
3. Each game we select should develop at least one prerequisite mathematics skill either directly or indirectly. For example, the commercially available game Master Mind is an excellent means for developing pattern recognition and visual memory and a good vehicle for developing deductive thinking in

children as well as adults.

Every teacher and parent has a favorite list of games. Some games might have been prepared or collected for a specific purpose—reinforcing a skill, teaching a concept, strengthening a process, or just offering entertainment.

Following is a list of games and toys I have used extensively with children and adults to help develop prerequisite skills and mathematics concepts and thinking. I have identified the prerequisite skills in each game. Most of these games and toys are commercial. They are highly motivational and can break formal instructional routines although they should not be used simply to occupy children's time. These games:

- have educational value
- are and should be fun
- are a natural activity in children's visual/perceptual development
- further their cognitive, affective and psycho-motoric development
- are useful assessment tools

List of Games:

- **Battleships (**spatial orientation, visualization, visual memory)
- **Black-Box** (logical deduction)
- **Blink** (pattern recognition, visual memory, classification, inductive reasoning)
- **British Squares** (spatial orientation, pattern recognition)
- **Card Games** (visual clustering, pattern recognition, number concept—visual clustering, decomposition/recomposition of number, number facts) (see Number War Games)
- **Checkers (**sequencing, patterns, spatial orientation/space organization)
- **Chinese Checkers** (patterns, spatial orientation/space organization)
- **Concentration** (visualization, pattern recognition, visual memory)
- **Cribbage (**number relationships, patterns, visual clusters)
- **Cross Number Puzzles** (number concepts, number facts)
- **Dominos (**visual clusters, pattern recognition, number concept and facts, decomposition/recomposition, number) (Number War Games)
- **Four Sight** (spatial orientation, pattern recognition, logical deduction)
- **Go Muko** (pattern recognition, spatial organization)
- **Go Make Ten** (**Go Fish Ten** or **Big Ten**) (number concept, decomposition/recomposition)

- **Hex** (pattern recognition)
- **In One Ear and Out the Other** (number relationships, number facts, additive reasoning)
- **Kalah**, **Mankalah,** or **Chhonka** (sequencing, counting, estimation, visual clustering, deductive reasoning)
- **Krypto** (number sense, basic arithmetical facts)
- **Math Bingo Games** (number facts)
- **Master Mind** (sequencing, logical deduction, pattern recognition)
- **Number Master Mind** (number concept, place value, properties of numbers)
- **Number Safari** (number facts, additive and multiplicative reasoning, equations, a paper/pencil game)
- **Number War Games** (visual clustering, arithmetic facts, mathematics concepts, deductive reasoning, fluency of facts)
- **Othello (**pattern recognition, spatial orientation, visual clustering, focus on more than one aspect, variable or concept at a time)
- **Parcheesi** (sequencing, patterns, number relationships)
- **Pinball Wizard** (number facts, a paper/pencil game)
- **Pyraos** (spatial orientation/space organization)
- **Quarto** (spatial orientation/space organization, patterns, classification)
- **Qubic** (pattern recognition, spatial orientation, visualization, geometrical patterns)
- **Reckon** (number facts, estimation, basic operations)
- **Score Four** or **Connect Four** (pattern recognition, spatial orientation, visual clustering, geometrical patterns)
- **Shut the Box (sequencing, number concept, and number facts)**
- **Simon** or **Mini Wizard** (sequencing, following multi-step directions, visual and auditory memory)
- **Snakes and Ladders** (sequencing, following multi-step directions, visualization, number facts)
- **Stratego** (spatial orientation, logical deduction, graphing)

Games should not simply be used to occupy children's time. Rather, they should be used as mathematics learning tools. Games should be used purposefully. Initially, all activities, games, software, or equipment must be teacher/parent directed and goal oriented. The involvement of the teacher/parent is essential for success and progress.

Engaging all children in a single game assumes that there are no individual differences among children, parents, or teachers. The key to the wise selection and use of games and toys is first to determine what prerequisite skills the child needs and then to select the appropriate games and toys.

The **Number War Games**, a collection of games, which I designed based on the popular game ***Game of War*** using ordinary deck of cards to teach number and number relationships is useful for developing arithmetic skills effectively and efficiently. These games use ordinary decks of playing cards and dominos—a versatile set of tools for teaching mathematics from number conceptualization to introductory algebra. In order that children develop better number concepts, it is better to play these games with Visual Cluster Cards (cards that do not have numbers on them, the cluster on the card represents the number). Cards without numbers on them are available from ***Center for Teaching/Learning of Mathematics.***

Number War Games begin in the same way as the Game of War. They are played essentially the same way and are easy to learn. To avoid the word war, you can call it by some other name such as: "beat it" or "top it." Children love to play these games. I have successfully used them for initial as well as remedial instruction, particularly for learning number, arithmetic facts, comparison of fractions, and operations on integers (treating club and spade cards as positive numbers and heart and diamond cards as negative numbers, for example, five of spades is $^{+}5$, and six of diamonds as $^{-}6$ and assign any value to face cards, e.g., Jack = 11, Queen = 12, King = 13, Jocker = variable value, Ace = 1). Once they master arithmetic facts with these cards, one should extend the idea to algebra (e.g., In this game, one with bigger value for $P = 2x + 3y$, where x is the value of the red card and y is the value of the black card. The expression for P changes ($P = x^2 + y^2$, $P = 2x/3y$, $P = |x| - 3|y|$, *etc.)* with each game (See *Number War Games*[2] for detailed instructions).

In addition to developing prerequisite skills, manipulative devices and games may be used in other ways:

- as a help in demonstrating the mathematical process,
- by children as they practice a process for ease in computation,
- by children as they practice to gain speed and accuracy in recall.

Furthermore, games and play provide good opportunities for discussions of strategies, outcomes, and feedback to improve strategies. Regular discussions invite children to communicate concepts while sharpening their thinking skills such as their ability to make inferences, to support their arguments with reasons, and to make analogies—skills essential to learning and applying mathematical skills.

In an environment where discussions are encouraged, children begin to ask questions not only of their classmates and of siblings but also of parents. They learn

[2] Available from *Center for Teaching/Learning of Mathematics*

to evaluate answers, draw conclusions, and follow up with more questions both of convergent (a question that calls for a yes, no or a short answer) and of divergent (a question that calls for an answer with explanation) types, which strengthen facility in reasoning.

Development and use of reasoning is the core of mathematics learning. Parents can do a lot in their children's education. Research shows that parental involvement—reading aloud, discussing the numbers children encounter in their environment, helping children to master arithmetic facts, checking homework, attending school meetings and events, setting expectations, relating current behavior with future accomplishments, and discussing school activities at home—has a more powerful influence on students' academic performance than anything about the school the students attend.

According to social science research, a major part of the academic advantage held by children from certain groups of families comes from well-organized, intentional, and concerted cultivation of children and their interests as compared to the more laissez-faire style of parenting common in most families. For this parents do not need to buy expensive educational toys, take children to enrichment classes, or explore digital devices for their kids in order to give them an edge in academics. It is discussion of ideas, exploring of interests, cultivating and supporting effort that develop children's interest and effort in learning.

The content and nature of these conversations and discussions also matters. Children who hear talk about counting and numbers at home start school with much more extensive mathematical knowledge—more number words, comparative words, and sizes of numbers, relating numbers, and combining and breaking numbers apart—knowledge that predicts future achievement in mathematics. Similarly, discussions about the spatial aspects of their world has impact on their understanding about the spatial properties of the physical world—how big or small or round, sharp objects, angles, or sides are. Both quantitative and spatial discussions have the impact on children's problem-solving abilities in future mathematics.

Without discussions, children become procedurally oriented. Too much procedural or 'recipe' learning eventually leads to boredom in mathematics. In contrast, the culture that inculcates mathematical thinking does not emphasize just formal mathematics learning but also develops skills in informal settings, which are forerunners of formal mathematical thinking.

To the uninitiated, mathematical objects are abstract, unreal, but for those who enjoy mathematics they are real, almost concrete objects. Doing real mathematics is like playing a game; it is thinking about and acting upon mathematical objects and the relationships among them, using the same mental abilities that we use to think about physical space, other people, or games and toys. To engage children in mathematics and excite them about mathematics learning, they need to see mathematics as a collection of interesting games and means of communication. This communication is enhanced when there is an intentional effort to talk about mathematics to children. Many of us feel completely comfortable talking about letters, words and sentences with our children—reading to them at night, talking about their games, toys, television programs, helping them decode their own books, noting messages on street signs and billboards.

Speaking to them about quantities (numbers, fractions, percents, decimals, greater, fewer, some, etc.), and shapes is not common. Some parents engage their children in "math talk or number talk." But, even they do not do so much and they do not do it that frequently. And yet research studies show that early "number talk" at home is a key predictor of young children's achievement in mathematics once they get to school. Research provides evidence that gender is also part of this phenomenon: Parents speak to their daughters about numbers far less than their sons.

A study by Alicia Chang and her colleagues (2010) published in the *Journal of Language and Social Psychology* drew on a collection of recordings of mothers talking to their toddlers, aged 20 to 27 months found that mothers spoke to their sons about number and number related concepts twice as often as they spoke to their daughters. Children this age are rapidly building their native language vocabularies, but, if encouraged, they can also build vocabularies for quantity and spatial concepts. These studies show that helping children become familiar with number and spatial concept words (to the left, above, below, next, farther, closer, bigger, higher, taller, longer, etc.) can promote their interest in mathematics as they enter school and in later grades.

The size of children's mathematics vocabulary, as in the case of native language, is dependent on exposure to the words in context. That was made clear in another study by psychologist Susan Levine and her colleagues (2010) published in *Developmental Psychology*, which also used recordings of parents talking to their children to gauge how often number words were used (the children in this study were between the ages of 14 and 30 months). They found huge variation among the families studied: Some children were hearing their parents speak only about two dozen number words a week, while others were hearing such words about 1,800 times weekly.

The frequency of number talk in the children's homes has a big impact on how well the youngsters understood basic mathematical concepts such as the cardinal number principle, which holds that the last number reached when counting a set of objects determines the size of the set ("One, two, three—three apples in the bowl!") and then the idea that a number is the property of the collection and not just the outcome of the counting process. Levine also found that the kind of number talk that most strongly predicted later knowledge of numbers involved counting or labeling sets of objects that are right there in front of parent and child–especially large sets, containing between four and ten objects.

Though it may not come naturally at first, parents can develop the habit of talking about numbers as often as they talk about letters and words. Some simple ways to work numbers into the conversation:

- Note numbers on signs when you're walking or driving with children: speed limits and exit numbers, building addresses, sale prices in store windows.
- Ask children to count how many toys they're playing with, how many books they've pulled out to read, or how many pieces of food are on their plate.
- Use numbers when you refer to time, dates, and temperatures: how many hours and minutes until bedtime, how many weeks and days until a holiday, the high and low the weatherman predicts for that day.
- With older children, math can become a part of talking about sports, science, history, video games, or whatever else they're interested in.

With practice, parents and children alike will find that math makes a very satisfying second language.

The Sequence of Strategies for Teaching Addition Facts[3]

0. Forty Five Sight facts using decomposition/recomposition 1 + 1; 1 +2;
1. Commutative Property (The turn around facts) (M + N = N + M) (*100* individual *facts are reduced to only 55 facts.)*
2. *N + 1, 1 + N (Adding one more to a number means getting the next number) [1+1; 1+2, 2+1; 3+1, 1+3; 1+4, 4+1; 5+1, 1+5; 1+6, 6+1; 1+7, 7+1; 8+1, 1+8; 9+1, 1+9; 10+1, 1+10]*
(19 facts on the Addition Grid: 10 on the first row and 9 on the first column in the grid.)
3. Making Ten (A pair of numbers that make 10?)
[2+8, 8+2; 3+7, 7+3; 4+6, 6+4; 5+5]
(7 new facts on the Addition Grid)
4. N + 10, 10 + N (The teens numbers)
[10+2, 2+10; 3+10, 10+3; 4+10, 10+4; 10+5, 5+10; 6+10, 10+6; 10+7, 7+10; 8+10, 10+8; 9+10, 10+9; 10+10]
(17 new facts on the Addition Grid)
5. N + 9, 9 + N (9 plus a number means adding 10 and minus 1)
[9+2, 2+9; 3+9, 9+3; 4+9, 9+4; 5+9, 9+5; 9+6, 6+9; 7+9, 9+7; 9+8, 8+9; 9+9] (15 new facts on the Addition Grid)
6. N + N (Double numbers)
[2+2, 3+3, 4+4, 6+6, 7+7,8+8]
(6 new facts on the Addition Grid)
7. N + (N+1) and (N + 1) + N (Near Doubles; If I know the double, I know one more than the double or one less than the double)
[2+3, 3+2; 3+4, 4+3; 4+5, 5+4; 5+6, 6+5; 6+7, 7+6; 7+8, 8+7]
(12 new facts on the Addition Grid)
8. N + (N − 2) (Adding Numbers that are two apart, then the their sum = double of the middle) [2+4, 4+2; 3+5, 5+3; 5+7, 7+5; 6+8, 8+6]
(8 new facts on the Addition Grid)
9. N + 2, 2 + N (Adding 2 to a number means skipping a number)
[5+2, 2+5; 6+2, 2+6; 7+2, 2+7]
(6 new facts on the Addition Grid)
10. Near Tens (The sum of two numbers, that is 1 more or 1 less than 10) [3+6, 6+3; 4+7, 7+4; 3+8, 8+3]
(6 new facts on the Addition Grid)
11. Remaining four facts
[8+4, 4+8; 8+5, 5+8]

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3 The most effective way of teaching Arithmetic facts is using Cuisenaire rods (see Cuisenaire rods and Mathematics Learning by Mahesh Sharma, 1988)

The Sequence Strategies for Teaching Multiplication Facts

1. Commutative Property (Turn around facts: M · N = N · M)
(The 100 individual facts of multiplication are reduced to only 55 facts.)
2. N · 1 = 1 · N (Table of 1: Multiplying a number by one results in the same number)
[1×1; 1×2, 2×1; 3×1, 1×3; 1×4, 4×1; 5×1, 1×5; 1×6, 6×1; 1×7, 7×1; 8×1, 1×8; 9×1, 1×9; 10×1, 1×10]
(19 individual facts on the Multiplication Grid)
3. N × 10, 10 × N (Table of 10 means Counting by 10)
[10×2, 2×10; 3×10, 10×3; 4×10, 10×4; 10×5, 5×10; 6×10, 10×6; 10×7, 7×10; 8×10, 10×8; 9×10, 10×9; 10×10]
(17 new individual facts on the Multiplication Grid)
4. N × 5, 5 × N (Table of 5 means counting by 5)
[5×2, 2×5; 5×3, 3×5; 5×4, 4×5; 5×5; 5×6, 6×5; 5×7, 7×5; 5×8, 8×5; 9×5, 5×9]
(15 new individual facts on the Multiplication Grid]
5. **N × 2, 2 × N (Table of 2 means doubles or counting by 2**)
[2×2; 2×3, 3×2; 2×4, 4×2; 2×6, 6×2; 2×7, 7×2; 2×8, 8×2; 9×2, 2×9]
(13 new individual facts on the Multiplication Grid)
6. N × 9, 9 × N
[3×9, 9×3; 4×9, 9×4; 9×6, 6×9; 7×9, 9×7; 9×8, 8×9; 9×9]
(11 new individual facts on the Multiplication Grid)
7. 4 × N, N × 4
[4×3, 3×4; 4×4; 4×6, 6×4; 4×7, 7×4; 8×4, 4×8]
(9 new individual facts on the Multiplication Grid)
8. N × N (Square numbers)
[3 × 3, 6 × 6, 7 × 7, 8 × 8]
(4 new facts on the Multiplication Grid)
9. N × 3, 3 × N (Table of 3)
[3 × 6, 6 × 3; 3 × 7, 7 × 3; 3 × 8, 8 × 3]
(6 new facts on the Multiplication Grid]
10. The remaining six facts [8 × 6, 6 × 8; 8 × 7, 7 × 8; 7 × 6, 6 × 7]
11. Distributive property: 12 × 8 = (10 + 2) × 8 = 10 × 8 + 2 × 8.

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Center for Teaching/Learning of Mathematics

CT/LM has programs and materials to assist teachers, parents, tutors, and diagnosticians to help children/adults with their learning difficulties in mathematics. We conduct regular **workshops, seminars,** and **lectures** on topics such as:

1. How does one learn mathematics? This workshop focuses on psychology and processes of learning mathematics—concepts, skills, and procedures. The role of factors such as: Cognitive development, language, mathematics learning personality, pre-requisite skills, conceptual models, and key developmental milestones (number conceptualization, place value, fractions, integers, algebraic thinking, and spatial sense) in mathematics learning. Participants learn strategies to teach their students more effectively.

2. What are the nature and causes of learning problems in mathematics? This workshop focuses on understanding the nature and causes of learning problems in mathematics. We examine existing research on diagnosis, remedial and instructional techniques in dealing with these problems. Participants become familiar with diagnostic and assessment instruments for learning problems in mathematics. They learn strategies for working more effectively with children and adults with learning problems in mathematics such as: dyscalculia and math anxiety.

3. Content workshops. These workshops are focused on teaching key mathematics milestone concepts and procedures. For example, **How to teach arithmetic facts easily and effectively. How to teach fractions more effectively. How to develop the concepts of algebra easily. Mathematics As a Second Language.** In these workshops, we use a new approach called **Vertical Acceleration.** In this approach, we begin with a simple concept from arithmetic and take it to the algebraic level.

4. What to look for in a results-oriented mathematics classroom: This is a workshop for administrators and teachers to understand the key elements necessary for an effective mathematics classroom.

We offer **individual diagnosis** and **tutoring services** for children and adults to help them with their mathematics learning difficulties, general learning problems, and dyscalculia. We provide:

1. Consultation with and training for parents and teachers to help their children cope with and overcome their anxieties and difficulties in learning mathematics, including dyscalculia.
2. Consultation services to schools and individual classroom teachers to help them evaluate their mathematics programs and teaching and help design new programs or supplement existing ones in order to minimize the incidence of learning problems in mathematics.
3. Assistance for the **adult student** who is returning to college and has anxiety about his/her mathematics.
4. Assistance in test preparation **(SSAT, SAT, GRE, GMAT, MCAS**, etc.)
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Professor Mahesh Sharma is the founder and President of the Center for Teaching/Learning of Mathematics, Inc., Framingham, Massachusetts, and Berkshire Mathematics in England. Berkshire Mathematics facilitates his work in the UK and Europe.

He is the former President and Professor of Mathematics Education at Cambridge College, where for more than thirty-five years, he taught mathematics and mathematics education to undergraduate and graduate students.

He is internationally known for his groundbreaking work in mathematics learning problems and mathematics education, particularly dyscalculia and other specific learning disabilities in mathematics. He is an author, teacher and teacher-trainer, researcher, consultant to public and private schools, as well as a public lecturer.

Professor Sharma was the Chief Editor and Publisher of *Focus on Learning Problems in Mathematic*s, an international, interdisciplinary research journal with readership in more than 90 countries, and the Editor of *The Math Notebook*, a practical source of information for parents and teachers devoted to improving teaching and learning for all children.

Professor Sharma provides direct services of evaluation and tutoring for students (children as well as adults) who have learning disabilities such as dyscalculia or face difficulties in learning mathematics and gifted/talented children to help them reach their potential. He works with teachers and school administrators to design strategies to improve mathematics curriculum and instruction for all. He has been a consultant to many educational organizations, many school systems, states and provinces in North America, and countries in Asia, and Europe.

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